

Dunstall Hill Primary School

Calculation Policy



The purpose of our Calculation Policy is to ensure consistency in the teaching of Mathematics throughout the school and to ensure that pupils develop efficient written and mental methods of calculation, underpinned by conceptual understanding.

Calculation Policy

This policy provides an overview of the strategies used in our school to teach Mathematics, specifically the four operations, as defined within the National Curriculum in England: Mathematics Programme of Study.





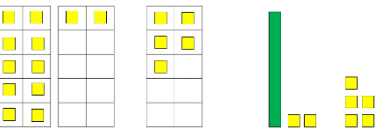
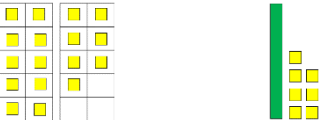

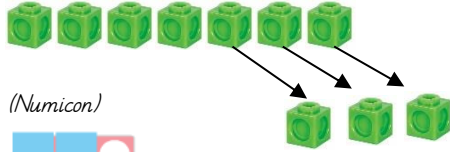


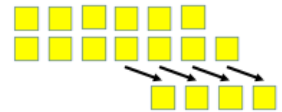
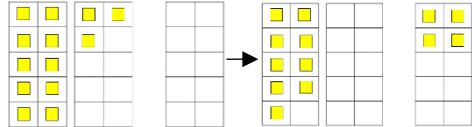



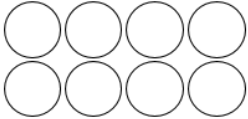



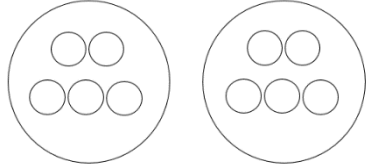
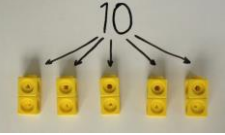
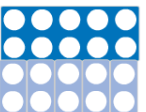
The progression of the four operations (+, -, x and \div) are shown across each of the primary year groups 1 - 6. This is a guide since children progress at different rates. Teachers should model strategies appropriate to the ability of the children they teach, regardless of their year group, whilst striving to achieve age related expectations at the end of the academic year.

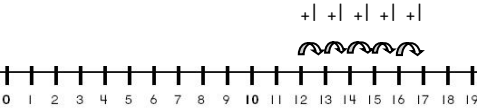
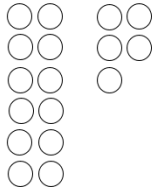

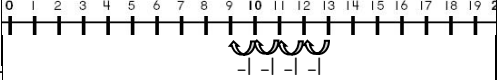
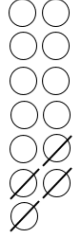

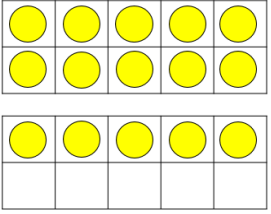


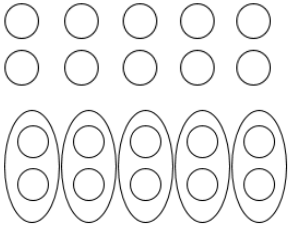

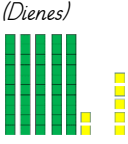
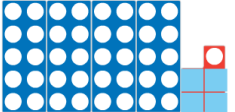
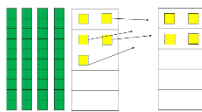

At Dunstall Hill Primary School, we believe that children should be introduced to the processes of calculation through the **concrete, pictorial** and **abstract** (CPA) approach. Our children are introduced to calculation through practical activities, using **concrete** resources. As children develop their understanding of the underlying concepts and mathematical models, they develop ways of recording to support their thinking. In the first instance, this recording takes the form of **pictorial** representations. Over time, children learn how to use models and images to support their mental and informal written methods of calculation.

As children become more proficient in their use of mental methods, their informal written methods also become more efficient. Some recording takes the form of jottings, which are used to support children's thinking. More **abstract**, formal written methods are taught only when the child is able to use a wide range of mental calculation strategies and these are always underpinned by **concrete** and **pictorial** experiences.

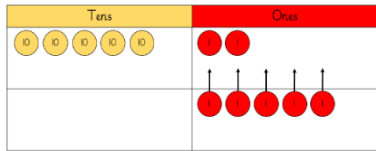
Our ultimate aim is for children to be able to select an efficient method to solve problems. Therefore children will be encouraged to look at a calculation or problem and to determine the most appropriate method to choose – pictures, mental calculation with or without jottings or a formal, written method.

The end of year expectations in the National Curriculum shows the progression in children's use of calculation within the following strands 'Addition and Subtraction' and 'Multiplication and Division'. These end of year expectations will be achieved through the use of the following written methods of calculation.

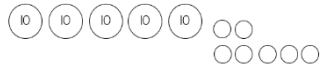
Year	Addition +	Subtraction -	Multiplication x	Division ÷
1	<ul style="list-style-type: none"> • Add one-digit and two-digit numbers to 20 including zero. • Read, write and interpret mathematical statements involving addition (+) and equal (=) signs. 	<ul style="list-style-type: none"> • Subtract one-digit and two-digit numbers to 20 including zero. • Read, write and interpret mathematical statements involving subtraction (-) and equal (=) signs. 	<ul style="list-style-type: none"> • Begin to understand multiplication through doubling numbers and quantities. • Use arrays and sets of 'equal groups' to look at other multiples, e.g. x 5. 	<ul style="list-style-type: none"> • Begin to understand division through grouping and sharing small quantities.
	<p>Addition of single digits: $5 + 3 = 8$... using concrete equipment:  (Numicon)  Addition of two digit numbers to 20 and a one digit number: $12 + 5 = 17$... using concrete equipment: (Numicon)  (Dienes)  (Dienes and ten frames)   (Bead string) </p>	<p>Subtraction of single digits $7 - 4 = 3$... using concrete equipment:  (Numicon)  Subtraction of a one-digit number from a two-digit number to 20. $13 - 4 = 9$... using concrete equipment: (Numicon)  (Dienes)  (Dienes and ten frames)  (Bead string) </p>	<p>Doubling – linking to x 2 $4 + 4 = 8$ or $4 \times 2 = 8$... using concrete equipment:  (Numicon)  ... using pictorial representations:  <i>Use an array or equal groups to solve multiplication problems for multiples other than 2</i> $5, 3 \text{ times or } 5 \times 3 = 15$... using concrete equipment (Numicon)  <i>I then use my 10s checker</i> </p>	<p>Sharing equally $10 \div 2 = 5$ Share 10 into 2 equal groups ... using concrete equipment:  <i>Count how many are in each set = 5</i> ... using pictorial representations:  ... using abstract number sentences: $10 \div 2 = 5$ Grouping How many 2s are in 10? What is 10 grouped into twos? ... using concrete equipment:  <i>Count how many groups = 5</i> (Numicon)  <i>Model putting the 2s on top of the ten Numicon tile. How many 2s have I used? 5</i></p>

	<p>... using pictorial representations:</p> <p>(Number line)</p>  <p>(Counters)</p>  <p>... using abstract mental strategies:</p> <p>(Counting on)</p> <p>"put 12 in your head and count on 5"</p> <p>13, 14, 15, 16, 17</p> 	<p>... using pictorial representations:</p> <p>(Number line)</p>  <p>(Counters)</p>  <p>... using abstract mental strategies:</p> <p>(Counting back)</p> <p>"put 13 in your head and count back 4"</p>  <p>12, 11, 10, 9</p>	<p>(Arrays, ten frames and counters)</p>  <p>... using pictorial representations:</p> <p>(Arrays)</p>  <p>... using abstract mental strategies:</p> <p>(Counting in multiples)</p> <p>5 10 15 or 2, 4, 6</p> 	<p>... using pictorial representations:</p>  <p>... using abstract number sentences:</p> <p>$10 \div 2 = 5$</p>
2	<p>• Add numbers, including:</p> <ul style="list-style-type: none"> a two-digit number and ones a two-digit number and tens two two-digit numbers adding three one-digit numbers <p>• Show that addition of two numbers can be done in any order (commutative).</p>	<p>• Subtract numbers, including:</p> <ul style="list-style-type: none"> a two-digit number and ones a two-digit number and tens two two-digit numbers <p>• Show that subtraction of two numbers cannot be done in any order.</p>	<p>• Calculate multiplication statements within the 2, 5 and 10 multiplication tables and write them using the multiplication (\times) and equals ($=$) signs.</p> <p>• Show that multiplication of two numbers can be done in any order (commutative).</p>	<p>• Calculate division statements within the 2, 5 and 10 multiplication tables and write them using the division (\div) and equals ($=$) signs.</p> <p>• Show that division of numbers cannot be done in any order.</p>
	<p>Addition of a two-digit number and ones:</p> <p>$52 + 5 = 57$</p> <p>... using concrete equipment:</p> <p>(Numicon)</p>  <p>(Dienes)</p> 	<p>Subtraction of a two-digit number and ones</p> <p>$45 - 4 = 41$</p> <p>... using concrete equipment:</p> <p>(Numicon)</p>  <p>(Dienes)</p> 	<p>Multiplication of two numbers within the 2, 3, 5, 10 multiplication tables.</p> <p>Introduce \times sign to mean 'how many times' and model recording calculations</p> <p>$5 \times 3 = 15$ or 5, 3 times = 15</p> <p>... using concrete equipment:</p> <p>(Numicon)</p> 	<p>Division of numbers within known multiplication tables</p> <p>Consolidate understanding of 'sharing' and 'grouping' as outlined within Year 1.</p> <p>Grouping</p> <p>How many 2s are in 10? What is 10 grouped into twos?</p> <p>... using concrete equipment:</p>

(Place value counters)



... using **pictorial** representations:

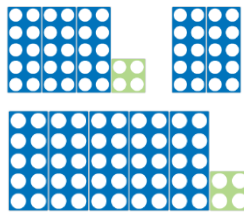


Addition of a two-digit number and tens

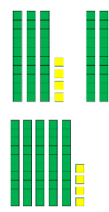
$$34 + 20 = 54$$

... using **concrete** equipment:

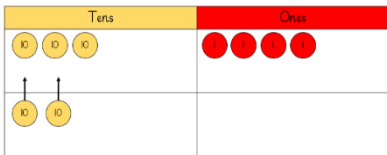
(Numicon)



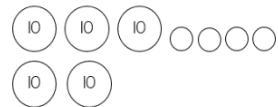
(Dienes)



(Place value counters)



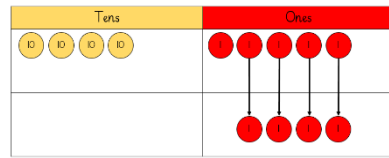
... using **pictorial** representations:



Addition of two two-digit numbers (no exchange):

$$34 + 23 = 57$$

(Place value counters)



... using **pictorial** representations:

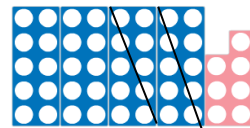


Subtraction of a two-digit number and tens

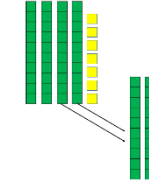
$$47 - 20 = 27$$

... using **concrete** equipment:

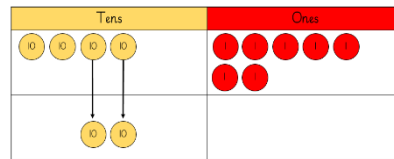
(Numicon)



(Dienes)



(Place value counters)



... using **pictorial** representations:

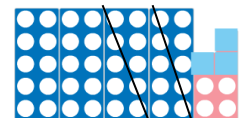


Subtraction two two-digit numbers (no exchange)

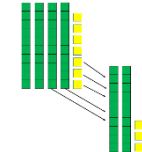
$$47 - 23 = 24$$

... using **concrete** equipment:

(Numicon)



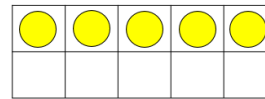
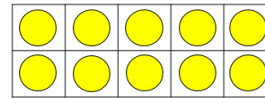
(Dienes)



I then use my 10s checker

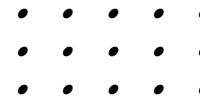


(Arrays, ten frames and counters)



... using **pictorial** representations:

(Arrays)

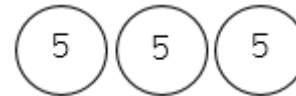


(Counters – one to many correspondence)

1) I need to write 5 out three times and count '1, 2, 3' as I do this.

5 5 5

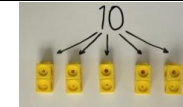
2) Now, I need to draw circles around my numbers as I count in multiple of 5. E.g. '5, 10, 15'



... using **abstract** mental strategies:

(Counting in multiples)

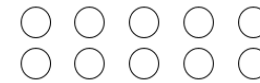
5 10 15 or 2, 4, 6 or 10, 20, 30



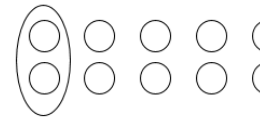
Count how many groups = 5
... using **pictorial**

representations:

1 2 3 4 5



1 2 3 4 5

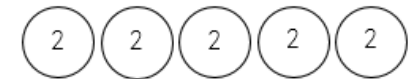


(Counters – one to many correspondence)

1) I need to write 2 as many times as it takes me to count in multiples of 2 to get to 10 e.g. 2, 4, 6, 8, 10.

2 2 2 2 2

2) Now, I need to draw circles around my numbers to count how many groups I have e.g. 1, 2, 3, 4, 5.



... using **abstract** number sentences:

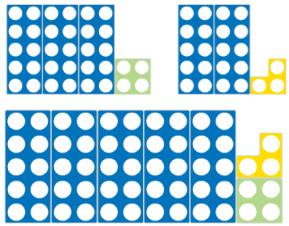
$$10 \div 2 = 5$$

$$12 \div 3 = 4$$

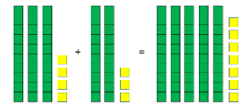
Pupils write number sentences to represent their workings out using the division (\div) and equals (=) signs.

... using **concrete** equipment:

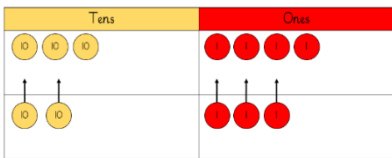
(Numicon)



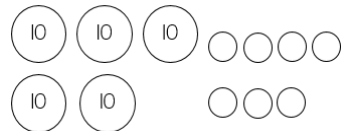
(Dienes)



(Place value counters)



... using **pictorial** representations:

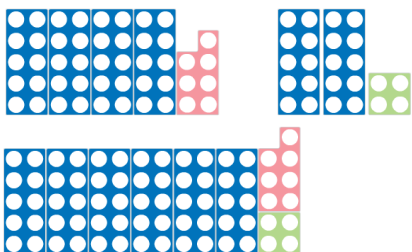


Addition of two two-digit numbers (exchange)

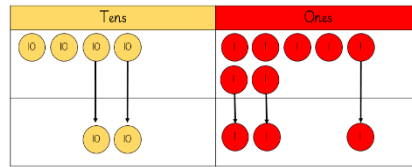
$$47 + 24 = 71$$

... using **concrete** equipment:

(Numicon)



(Place value counters)



... using **pictorial** representations:

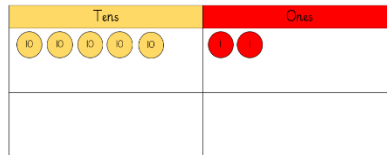


Subtraction of two two-digit numbers
(exchange)

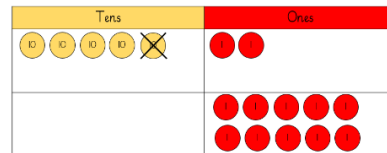
$$52 - 27 = 25$$

... using **concrete** equipment:

(Place value counters)



7 ones cannot be subtracted from 2 ones so
exchange 1 ten with 10 ones.



Now, subtract 7 ones.



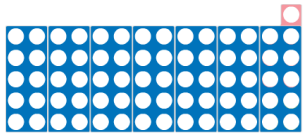
Calculate mathematical statements within the
2, 5 and 10 multiplication tables and write
them using the multiplication (\times) and equals
($=$) signs.

$$4 \times 5 = 20$$

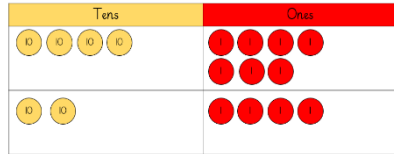
$$7 \times 10 = 70$$

$$9 \times 2 = 18$$

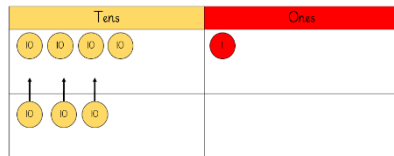
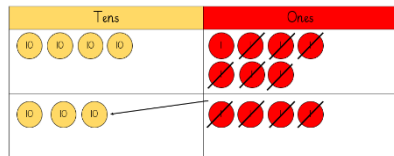
Then, I use my 10 checker.



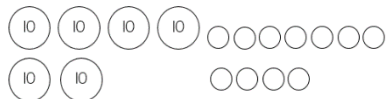
(Place value counters)



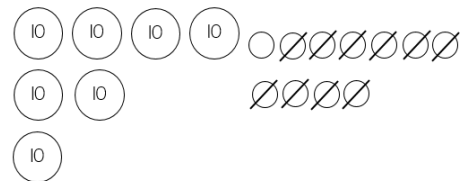
Because I have more than 9 ones, I need to exchange 10 ones for 1 ten.



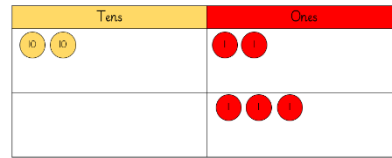
... using pictorial representations:



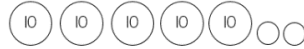
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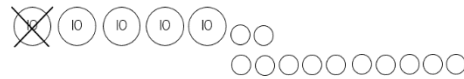
Now, subtract 2 tens.



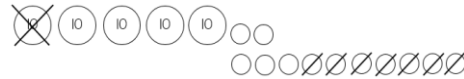
... using pictorial representations:



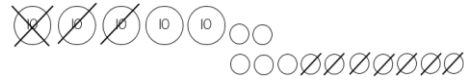
7 ones cannot be subtracted from 2 ones so exchange 1 ten with 10 ones.



Now, subtract 7 ones.



Now, subtract 2 tens.



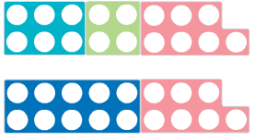
Following the **concrete** equipment and **pictorial** representations, children will use **abstract**, mental strategies:

$$45 - 4 = 41$$

$$47 - 20 = 27$$

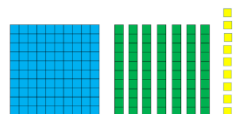
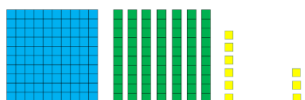
$$47 - 23 = 24$$

$$52 - 27 = 25$$

	<p>Following the concrete equipment and pictorial representations, children will use abstract mental strategies:</p> <p> $52 + 5 = 57$ $34 + 20 = 54$ $34 + 23 = 57$ $47 + 24 = 71$ </p> <p>Addition of three single digit numbers:</p> $4 + 7 + 6 = 17$ <p>... using concrete equipment:</p> <p>Identify number bonds if possible, e.g. 4 and 6 make 10 / $4 + 6 = 10$. Then, add on 7 (Numicon)</p>  <p>... using abstract, mental strategies:</p> $\textcircled{4} + 7 + \textcircled{6} = 17$ <p>Identify the two numbers that make ten and then add on the remaining number mentally.</p>			
3	<ul style="list-style-type: none"> * Add numbers mentally, including: <ul style="list-style-type: none"> ▪ a three-digit number and ones ▪ a three-digit number and tens ▪ a three-digit number and hundreds * Add numbers with up to three digits, using formal written methods of columnar addition 	<ul style="list-style-type: none"> * Subtract numbers mentally, including: <ul style="list-style-type: none"> ▪ a three-digit number and ones ▪ a three-digit number and tens ▪ a three-digit number and hundreds * Subtract a two-digit or 3-digit number from a two-digit or 3 digit number using a formal written method 	<ul style="list-style-type: none"> * Recall and use multiplication facts for the 3, 4 and 8 multiplication tables. * Multiply using multiplication tables that they know, including for two-digit numbers times one-digit numbers, using efficient written methods- 'partitioning method' 	<ul style="list-style-type: none"> * Recall and use division facts for the 3, 4 and 8 multiplication tables. * Divide using known multiplication tables, including for two-digit numbers divided by one-digit numbers, using mental methods, progressing to efficient written methods
	<p>Addition of a three-digit number and ones:</p> $176 + 3 = 179$	<p>Subtraction of a three-digit number and ones:</p> $136 - 4 = 132$	<p>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.</p> $8 \times 4 = 32$	<p>Recall and use division facts for the 3, 4 and 8 multiplication tables.</p> $56 \div 8 = 7$

... using **concrete** equipment:

(Dienes)



(Place value counters)

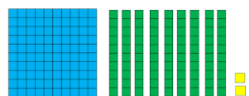
Hundreds	Tens	Ones
●	●●●●	●●

Addition of a three-digit number and tens:

$$342 + 50 = 392$$

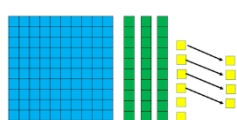
... using **concrete** equipment:

(Dienes)



... using **concrete** equipment:

(Dienes)



(Place value counters)

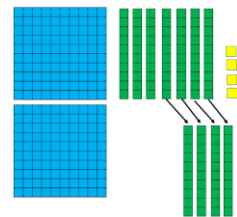
Hundreds	Tens	Ones
●	●●●●●●	●●●

Subtraction of a three-digit number and tens:

$$273 - 40 = 233$$

... using **concrete** equipment:

(Dienes)



(Place value counters)

Hundreds	Tens	Ones
●●	●●●●	●●●

Subtraction of a three-digit number and hundreds:

$$324 - 200 = 124$$

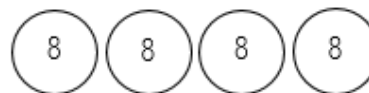
... using **pictorial** representations:

(Counters – one to many correspondence)

1) I need to write 8 out four times and count '1, 2, 3, 4' as I do this.

8 8 8 8

2) Now, I need to draw circles around my numbers and count in multiple of 8. E.g. '8, 16, 24, 32'



... using **abstract** mental strategies:

(Counting in multiples)

3, 6, 9... or 4, 8, 12... or 8, 12, 16...

Multiplication of a two-digit number by a one-digit number.

$$13 \times 4 = 52$$

$$24 \times 3 = 72$$

... using **concrete** equipment:

(Dienes)

Hundreds	Tens	Ones
	■■■■	■■■■

Count the number of ones, and then count the number of tens.

Hundreds	Tens	Ones
	■■■■	■■■■
	■■■■	■■■■
	■■■■	■■■■
	40	12

$$40 + 12 = 52$$

... using **pictorial** representations:

(Counters – one to many correspondence)

1) I need to write 8 as many times as it takes me to count in multiples of 8 to get to 56 e.g. 8, 16, 24, 32, 40, 48, 56.

8 8 8 8 8 8 8

2) Now, I need to draw circles around my numbers to count how many groups I have e.g. 1, 2, 3, 4, 5, 6, 7.



Division of a two-digit number by a one-digit number, using known multiplication tables.

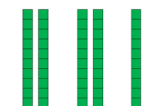
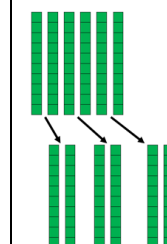
$$60 \div 3 = 20$$

... using **concrete** equipment:

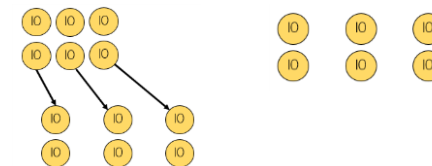
Sharing

Grouping

(Dienes)



(Place value counters)

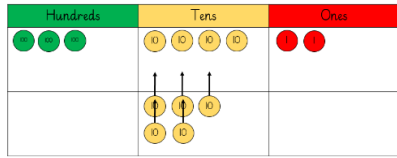


... using **abstract** mental strategies:

$$6 \text{ tens} \div 3 = 2 \text{ tens} = 20$$

Dividing a two-digit numbers by one-digit numbers.

(Place value counters)

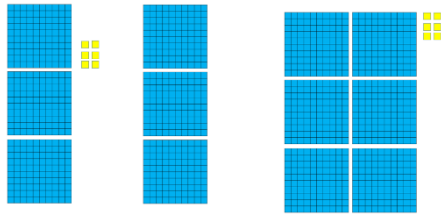


Addition of a three-digit number and hundreds:

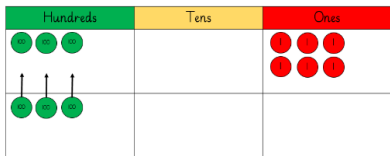
$$306 + 300 = 606$$

... using **concrete** equipment:

(Dienes)



(Place value counters)



Addition of numbers with up to three digits

$$263 + 129 = 392$$

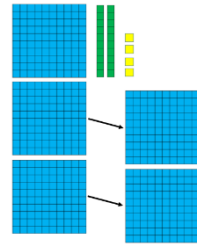
... using **concrete** equipment:

(Dienes)

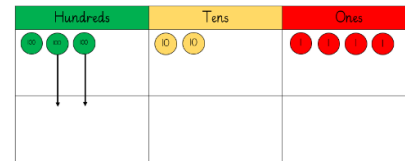
Thousands	Hundreds	Tens	Ones

... using **concrete** equipment:

(Dienes)



(Place value counters)



Subtraction of numbers with up to three digits

$$263 - 129 = 134$$

... using **concrete** equipment:

(Dienes)

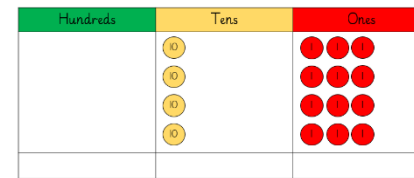
Thousands	Hundreds	Tens	Ones

9 ones cannot be subtracted from 3 ones so exchange 1 ten for 10 ones.

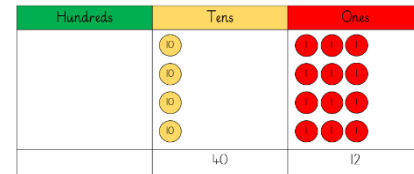
Thousands	Hundreds	Tens	Ones

(Place value counters)

First calculation

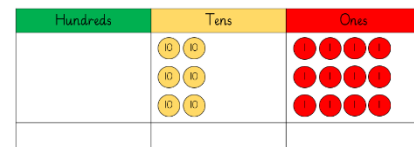


Count the number of ones, and then count the number of tens.

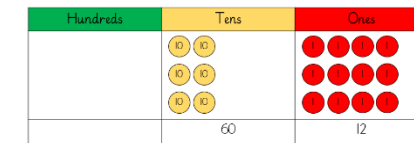


$$40 + 12 = 52$$

Second calculation



Count the number of ones, and then count the number of tens.



$$60 + 12 = 72$$

... using **pictorial** representations:

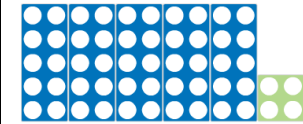
First calculation

Count the ones first, then the tens and add the numbers together.

$$54 \div 3 = 18$$

... using **concrete** equipment:

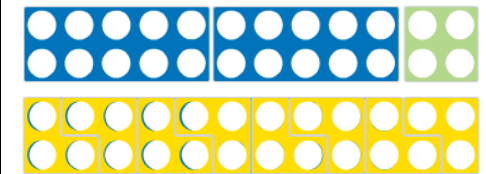
(Numicon)



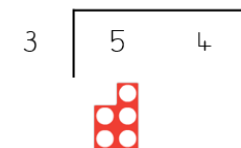
Share the tens equally into 3 groups.



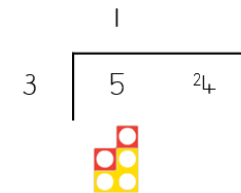
I have 24 left over. Now I need to divide 24 by 3.



(Numicon)



How many 3s goes into 5?



Exchange 10 ones for 1 ten.

Thousands	Hundreds	Tens	Ones
	2	10	2
	1	10	
			2

Thousands	Hundreds	Tens	Ones
	2	10	2
	1	10	
		9	2

Thousands	Hundreds	Tens	Ones
	3	9	2

(Place value counters)

Thousands	Hundreds	Tens	Ones
	2	10	2
	1	10	
			2

Exchange 10 ones for 1 ten.

Thousands	Hundreds	Tens	Ones
	2	10	2
	1	10	
			2

Now, subtract 9 ones.

Thousands	Hundreds	Tens	Ones
	2	10	2
			4

Now, subtract 2 tens.

Thousands	Hundreds	Tens	Ones
	2	10	2
		3	4

Now, subtract 1 hundred.

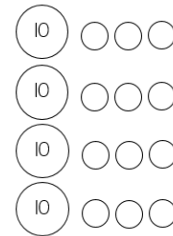
Thousands	Hundreds	Tens	Ones
	1	3	4

(Place value counters)

Hundreds	Tens	Ones
2	10	2
1	3	4

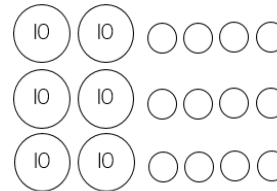
9 ones cannot be subtracted from 3 ones so exchange 1 ten for 10 ones.

Hundreds	Tens	Ones
2	10	2
	10	2
		2



$$40 + 12 = 52$$

Second calculation



$$60 + 12 = 72$$

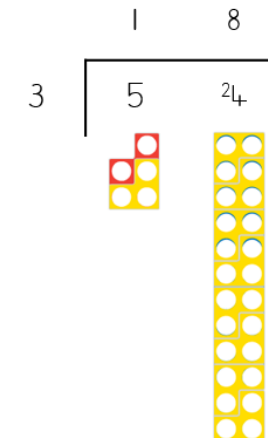
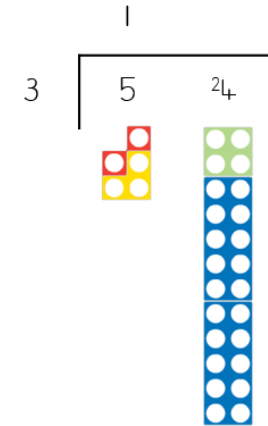
... using **abstract** methods:

Use of partitioning method, independent of equipment and diagrams.

$$\begin{aligned} 13 \times 4 &= (10 \times 4) + (3 \times 4) \\ &= 40 + 12 \\ &= 52 \end{aligned}$$

$$\begin{aligned} 24 \times 3 &= (20 \times 3) + (4 \times 3) \\ &= 60 + 12 \\ &= 72 \end{aligned}$$

Now, make 24 and check how many 3s go into 24.



... using **abstract** methods:

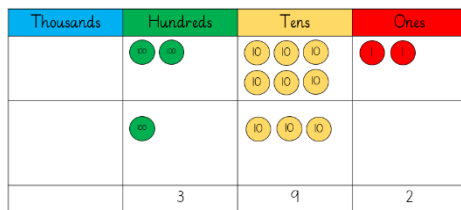
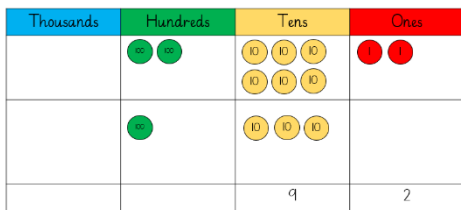
Completion of number sentences.

$$60 \div 3 = 20$$

Progression in the formal written method for division:

Step 1

Two-digit number divided by a one-digit number – no exchanging across place value columns e.g. $84 \div 4 = 21$



... using **pictorial** representations:



Exchange ten ones for 1 ten.



... using **abstract** mental strategies:

(Column method)

$$\begin{array}{r}
 263 \\
 + 129 \\
 \hline
 392 \\
 \hline
 1
 \end{array}$$

Progression in columnar addition:

Step 1 (to introduce)

2 digits - no exchanging e.g. $45 + 32$

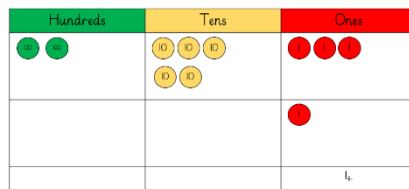
Step 2

2 digits - exchanging to the tens e.g. $43 + 18$

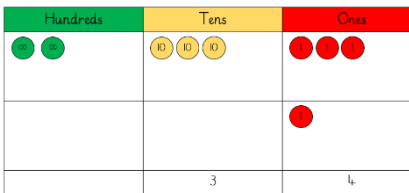
Step 3

3 digits - exchanging to the tens e.g. $263 + 119$

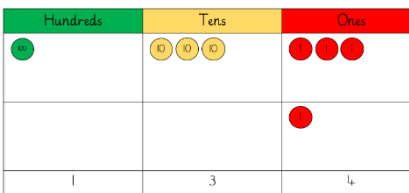
Now, subtract 9 ones.



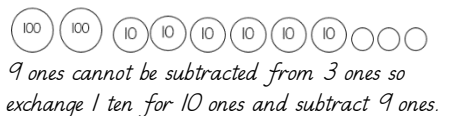
Now, subtract 2 tens.



Now, subtract 1 hundred.



... using **pictorial** representations:



9 ones cannot be subtracted from 3 ones so exchange 1 ten for 10 ones and subtract 9 ones.



Now, subtract 2 tens.



Now, subtract 1 hundred.



$$\begin{array}{r}
 21 \\
 4 \overline{) 84} \\
 \hline
 4 \\
 \hline
 4 \\
 \hline
 0
 \end{array}$$

Step 2

Two-digit number divided by a one-digit number - involving exchanging across place value columns without remainders e.g.

	1	8
3	5	24

	<p>Step 4 3 digits - exchanging to the hundreds e.g. $357 + 261$</p> <p>Step 5 3 digits - exchanging to the thousands e.g. $847 + 931$</p> <p>Step 6 2 and 3 digit numbers – understand place value including the place value of columns.</p>	<p>... using abstract mental strategies: (<i>Column method</i>)</p> $\begin{array}{r} 5 \quad 1 \\ 2 \quad \cancel{6} \quad 3 \\ - \quad 1 \quad 2 \quad 9 \\ \hline 1 \quad 3 \quad 4 \end{array}$ <p>Progression in columnar subtraction:</p> <p>Step 1 (to introduce) 2 digits - no exchanging e.g. $58 - 27$</p> <p>Step 2 2 digits - exchanging from tens e.g. $42 - 18$</p> <p>Step 3 3 digits - exchanging from tens e.g. $263 - 119$</p> <p>Step 4 3 digits - exchanging from hundreds e.g. $347 - 261$</p> <p>Step 5 2 from 3 digit numbers – understand place value including the place value of columns.</p>		
4	<ul style="list-style-type: none"> • Add numbers with up to 4 digits using mental strategies and the formal written methods (<i>columnar addition</i>) • Add numbers with 2 decimal places, using formal written methods (<i>columnar addition</i>) 	<ul style="list-style-type: none"> • Subtract numbers with up to 4 digits using mental strategies and the formal written methods (<i>columnar subtraction</i>) • Subtract numbers with 2 decimal places, using formal written methods (<i>columnar subtraction</i>) 	<ul style="list-style-type: none"> • Recall multiplication facts for multiplication tables up to 12 x 12. • Multiply two-digit and three-digit numbers by a one-digit number using formal written layout e.g. 84×6, 216×4 • Multiply three-digit numbers with 1 decimal place by a one-digit number using formal written layout e.g. 134.5×7 	<ul style="list-style-type: none"> • Recall division facts for multiplication tables up to 12 x 12. • Divide numbers up to 3 digits by a 1 digit number using the formal written method (no remainders)
	Addition of numbers with up to four digits:	Subtraction of numbers with up to four digits	Recall and use multiplication facts for the multiplication tables up to 12 x 12.	Recall and use division facts for the multiplication tables up to 12 x 12.

... using **concrete** equipment:

Use of place value chart and dienes (as used in Year 3).

Thousands	Hundreds	Tens	Ones

Use of place value chart and place value counters (as used in Year 3).

Thousands	Hundreds	Tens	Ones

... using **pictorial** representations:

Use of place value counters to support understanding (as used in Year 3).

... using **abstract** strategies:
(Column method)

four digit + four digit

$$\begin{array}{r} 4478 \\ + 3762 \\ \hline 8240 \\ \hline | \quad | \quad | \end{array}$$

... using **concrete** equipment:

Use of place value chart and dienes (as used in Year 3).

Thousands	Hundreds	Tens	Ones

Use of place value chart and place value counters (as used in Year 3).

Thousands	Hundreds	Tens	Ones

... using **pictorial** representations:

Use of place value counters to support understanding (as used in Year 3).

... using **abstract** strategies:

four digit – four digit

$$\begin{array}{r} 531 \\ - 2684 \\ \hline 3783 \end{array}$$

four digit – three digit

Understanding place value and the place value of columns

$$\begin{array}{r} 1431 \\ - 876 \\ \hline 1667 \end{array}$$

... using **concrete** equipment:

Use of counters – one to many correspondence (as used in Year 3).

... using **pictorial** representations:

Use of counters – one to many correspondence (as used in Year 3).

... using **abstract** mental strategies:

Counting in multiples (the same as year 3 but involving all multiplication facts up to 12 x 12)

Multiplication of two and three digit numbers by a one-digit number

$$216 \times 4 = 864$$

... using **concrete** equipment:

(Place value counters)

Thousands	Hundreds	Tens	Ones
	●●	●●	●●●●●●●●
	●●	●●	●●●●●●●●
	●●	●●	●●●●●●●●
	●●	●●	●●●●●●●●

First, count how many ones there are. Pupils to count in multiples e.g. 6, 12, 18, 24. Because I have '24' ones in one place value column, I know I need to exchange 20 ones for 2 tens and count how many ones are left.

Thousands	Hundreds	Tens	Ones
	●●	●●●●	●●●●●●●●
	●●	●●	●●●●●●●●
	●●	●●	●●●●●●●●
	●●	●●	●●●●●●●●

Now, count how many tens there are.

Thousands	Hundreds	Tens	Ones
	●●	●●●●	●●●●
	●●	●●	
	●●	●●	
	●●	●●	

... using **concrete** equipment:

Use of counters – one to many correspondence (as used in Year 3).

... using **pictorial** representations:

Use of counters – one to many correspondence (as used in Year 3).

... using **abstract** mental strategies:

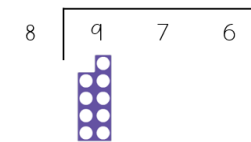
Counting in multiples (the same as year 3 but involving all division facts up to 12 x 12)

Divide numbers with up to three-digit by a one-digit number

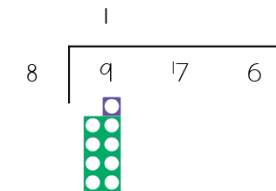
$$976 \div 8 = 122$$

... using **concrete** equipment:

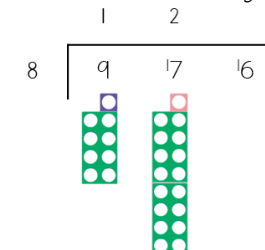
(Numicon)



How many 8s go into 9?



Now, make 17 and check how many 8s go into 17.



four digit + three digit

Understanding place value and the place value of columns

$$\begin{array}{r} 1456 \\ + 765 \\ \hline 2221 \\ \hline 111 \end{array}$$

Using 0 as a place holder

$$\begin{array}{r} 2605 \\ + 809 \\ \hline 3414 \\ \hline 11 \end{array}$$

Numbers with 1 decimal place

$$\begin{array}{r} 379.3 \\ + 203.5 \\ \hline 582.8 \\ \hline 1 \end{array}$$

Numbers with 2 decimal places

$$\begin{array}{r} 379.34 \\ + 203.52 \\ \hline 582.86 \\ \hline 1 \end{array}$$

*Use partitioning methods to support understanding of columnar addition where appropriate.

Using 0 as a place holder

$$\begin{array}{r} 591 \\ 2\cancel{0}\cancel{0}5 \\ - 89 \\ \hline 2516 \end{array}$$

Method 1

$$\begin{array}{r} 199 \\ \cancel{2}\cancel{0}\cancel{0}10 \\ - 475 \\ \hline 1525 \end{array}$$

Method 2

More efficient – Subtract 1 from both numbers in the calculation. $1999 - 474$

$$\begin{array}{r} 1999 \\ + 474 \\ \hline 1525 \end{array}$$

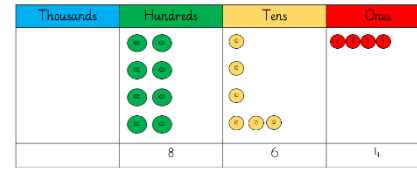
Numbers with 1 decimal place

$$\begin{array}{r} 31 \\ 7\cancel{0}3.7 \\ - 216.2 \\ \hline 527.5 \end{array}$$

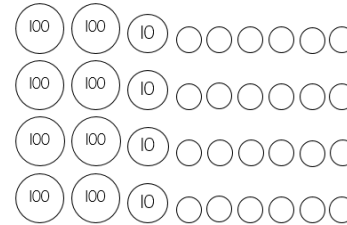
Numbers with 2 decimal places

$$\begin{array}{r} 31 \\ 7\cancel{0}3.72 \\ - 216.21 \\ \hline 527.51 \end{array}$$

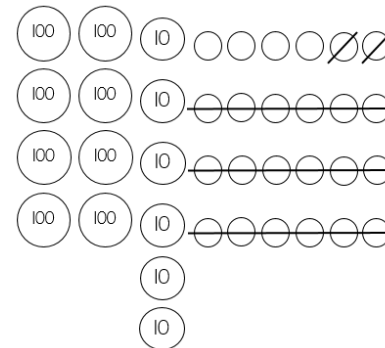
Now, count how many hundreds there are. Pupils to count in multiples. E.g. '2, 4, 6, 8'



... using pictorial representations:

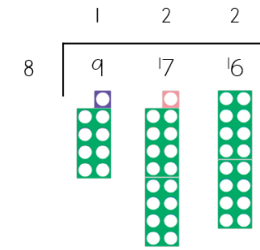


First, count how many ones there are. Pupils to count in multiples e.g. 6, 12, 18, 24. Because I know I cannot have '24' ones in one place value column, I know I need to exchange 20 ones for 2 tens and count how many ones are left.



Now, count how many tens there are and how many hundreds there are. Pupils to count in multiples e.g. 2, 4, 6, 8.

Now, make 16 and check how many 8s go into 16.



... using abstract methods:

Progression in the formal written method for division:

Step 1

Two and three-digit numbers divided by a one-digit number – no exchanging across place value columns e.g. $84 \div 4 = 21$, $396 \div 3 = 132$

$$\begin{array}{r} 21 \\ 4 \overline{) 84} \\ \underline{8} \\ 0 \end{array} \quad \begin{array}{r} 132 \\ 3 \overline{) 396} \\ \underline{3} \\ 0 \\ 0 \end{array}$$

Step 2

Two and three-digit numbers divided by a one-digit number – involving exchanging across place value columns without remainders e.g. $138 \div 6 = 23$, $976 \div 8 = 122$

$$\begin{array}{r} 23 \\ 6 \overline{) 138} \\ \underline{12} \\ 18 \\ \underline{18} \\ 0 \end{array} \quad \begin{array}{r} 122 \\ 8 \overline{) 976} \\ \underline{8} \\ 17 \\ \underline{16} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

* Introduce the concept of a remainder.

**Use partitioning methods to support understanding of columnar subtraction where appropriate.*

... using **abstract** methods:

Progression in column multiplication:

Step 1 (to introduce)

two digits x one digit - no exchanging e.g. 32 x 3

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

Step 2

two digits x one digit - exchange to tens e.g. 23 x 4

(Expand to model exchanging)

**Sometimes new arrivals arrive knowing the expanded version*

$$\begin{array}{r} 23 \\ \times 4 \\ \hline 92 \\ + 100 \\ \hline 112 \end{array}$$

Step 3

two digits x one digit - exchange to tens and hundreds e.g. 84 x 6

$$\begin{array}{r} 84 \\ \times 6 \\ \hline 504 \\ + 500 \\ \hline 504 \end{array}$$

			<p>Step 4</p> <p>three digits x one digit – exchange to tens e.g. 219×4</p> $\begin{array}{r} 219 \\ \times 4 \\ \hline 876 \\ \hline 3 \end{array}$ <p>Step 5</p> <p>three digits x one digit – exchange to tens, hundreds and thousands e.g. 425×4</p> $\begin{array}{r} 425 \\ \times 4 \\ \hline 1800 \\ \hline 122 \end{array}$	
5	<ul style="list-style-type: none"> • Add whole numbers with more than 4 digits (and with up to 3 decimal places), including using formal written methods (columnar addition) 	<ul style="list-style-type: none"> • Subtract whole numbers with more than 4 digits (and with up to 3 decimal places), including using formal written methods (columnar subtraction) 	<ul style="list-style-type: none"> • Multiply numbers up to 4 digits by a 1 digit number using a formal written method e.g. 3721×7 • Multiply one-digit numbers with up to three decimal places by whole numbers • Multiply numbers up to 4 digits by 2-digit number using a formal written method e.g. 3721×37 	<ul style="list-style-type: none"> • Divide numbers up to 4 digits by a one-digit number using the formal written method and interpret remainders • Divide numbers up to 4 digits with up to 3 decimal places by a one-digit number using the formal short written method
	<p><i>The same as Year 4 but with larger numbers and with a greater number of decimal places – up to 3 decimal places.</i></p> <p><i>Continue to ensure that the use of '0' as a placeholder is used to ensure pupils are confident with the exchanging and adding on process.</i></p>	<p><i>The same as Year 4 but with larger numbers and with a greater number of decimal places – up to 3 decimal places.</i></p> <p><i>Continue to ensure that the use of '0' as a placeholder is used to ensure pupils are confident with the exchanging process.</i></p>	<p>Multiplication of a four-digit numbers by a one-digit numbers.</p> <p>... using concrete equipment: <i>Use of place value counters (as used in Year 4).</i></p> <p>... using pictorial representations: <i>Use of place value counters (as used in Year 4).</i></p> <p>... using abstract methods:</p> $\begin{array}{r} 3721 \\ \times 7 \\ \hline 26047 \\ \hline 251 \end{array} \quad \begin{array}{r} 4725 \\ \times 9 \\ \hline 42525 \\ \hline 4624 \end{array}$	<p>Division of numbers with up to four digits by a one-digit number.</p> <p><i>Consolidate understanding of using the formal written method without remainders as outlined within Year 4.</i></p> <p>... using concrete equipment: <i>Use of Numicon (as used in Year 4)</i></p>

Multiplication of a one-digit number with up to three decimal places by a one-digit number.

$$\begin{array}{r} 1.43 \\ \times 600 \\ \hline 8.58 \\ 2 \quad 1 \end{array}$$

Develop to up to 4 digits with up to 3 decimal places by a one-digit number.

Multiplication of a four-digit number by a two-digit number.

$$\begin{array}{r} 3701 \\ \times 25907 \\ \hline 25907 \\ 259070 \\ 2590700 \\ 25907000 \\ \hline 136937 \end{array}$$

... using **abstract** methods:

Progression in the formal written method for division:

Step 1

Two-digit number divided by one-digit number – with remainders

$$76 \div 6 = 12 \text{ r } 4$$

	1	2	r	4
6	7	6		

Step 2

Three-digit number divided by one-digit number – with remainders

$$852 \div 7 = 121 \text{ r } 5$$

Round up or down given the context of the problem.

	1	2	1	r	5
7	8	5	2		

Step 3

Up to four-digits with up to 3 decimal places by a one-digit number

	2	4	.	9
7	1	7	4	3

	2	3	.	2	9
8	1	8	6	3	2

				<p>Step 4</p> <p>Four-digit number divided by one-digit number – with remainders- interpreted as a decimal (to 3 decimal places)</p> <p>$6497 \div 8 = 812.125$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td></td><td>0</td><td>8</td><td>1</td><td>2</td><td>.</td><td>1</td><td>2</td><td>5</td></tr> <tr> <td>8</td><td>6</td><td>4</td><td>9</td><td>7</td><td>.</td><td>0</td><td>2</td><td>0</td></tr> </table>		0	8	1	2	.	1	2	5	8	6	4	9	7	.	0	2	0
	0	8	1	2	.	1	2	5														
8	6	4	9	7	.	0	2	0														
6	<p>* Add multi-digit numbers with more than 4 digits (with up to 3 decimal places), using formal written methods (columnar addition)</p>	<p>* Subtract multi-digit numbers with more than 4 digits (with up to 3 decimal places), using formal written methods (columnar subtraction)</p>	<p>* Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</p>	<p>* Divide numbers up to 4 digits (with up to 3 decimal places) by a two-digit whole number using the formal written method of division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p> <ul style="list-style-type: none"> - Short division - Long division 																		
	<p><i>The same as Year 4 and 5 but with multi-digit numbers with more than 4 digits (and with up to 3 decimal places).</i></p>	<p><i>The same as Year 4 and 5 but with multi-digit numbers with more than 4 digits (and with up to 3 decimal places).</i></p>	<p>Multiplication of a four-digit number by a two-digit number.</p> $ \begin{array}{r} 3701 \\ \times 37 \\ \hline 25907 \\ 2 \quad 4 \\ + 111030 \\ \hline 136937 \end{array} $	<p><i>Consolidate understanding of using the formal written method for dividing three-digit number with up to 3 decimal places by one-digit number as outlined in Year 5.</i></p> <p>Division of numbers with up to four-digits and three decimal places, by a two-digit whole number.</p> <p>$4138 \div 17 = 243 \text{ r } 7$</p> <p>... using concrete equipment:</p> <p><i>Use of Numicon (as used in Year 4 and Year 5)</i></p>																		

